Transit Timetable Synchronization: Evaluation and Optimization

By Charles Fleurent, Réjean Lessard, and Lise Séguin

Introduction

Trip synchronization is an important issue for most public transit companies. Minimizing waiting times for transfers between trips can improve customer satisfaction, which in turn leads to increases in ridership and revenues. Several papers have been published on this subject in recent years. In most cases (see [2], [5]), researchers have studied straightforward mathematical formulations designed to generate timetables with maximal synchronization between trips and have developed heuristic optimization methods for the non-linear models that arise in this context. Although some models consider other factors such as headway regularization and trip frequencies, very few (see [3]) address more complex issues such as the impact of these timetables on the overall vehicle scheduling process.

One goal of this paper is to present a more elaborate measure of synchronization so it reflects the real concerns of schedulers in practice. Extensive discussions with representatives from public transport authorities based in Montréal, Canada (STM and AMT) led us to include in our model, several parameters such as minimum, maximum, and ideal waiting times, as well as weight factors specifying the relative importance of synchronization with respect to different times, places, routes, and directions. Once this data is provided to a system, it can be used to calculate a composite quality index for synchronization that reflects the objectives of the company. For instance, generating transfers that are close to ideal waiting times may be more important than minimizing overall waiting time, while transfer opportunities during peak hours or for the last trip on a given route may have a much larger impact on schedule quality than other transfers.

Another goal of this paper is to propose an optimization approach that embeds trip synchronization in a global algorithm that can also minimize other vehicle scheduling costs. In practice, fleet size, unproductive time (layovers and deadheads), and deviations to constraints must be considered so a trip timetable that generates improvements to synchronization can also lead to an efficient vehicle schedule. In our approach, an initial set of trips is used to generate a network in which each arc represents possible earlier or later shifts to the planned start time of a pair of trips that can be covered by the same vehicle. A heuristic algorithm then solves a series of network flow problems in which the cost of each arc includes both a synchronization quality index and other costs. The process generates an optimized vehicle schedule that puts more or less emphasis on synchronization, depending on parameters that apply relative weights to the multiple factors that should be considered.

The remainder of this paper is organized as follows. In the next section, some basic concepts are presented, as well as a timetable synchronization measure. In Section 3, a set of interactive computerized tools to analyze and possibly improve the synchronization of an existing timetable is described. Section 4 presents a global algorithmic approach that can both optimize an existing timetable to maximize synchronization, and generate a cost-effective vehicle schedule. Section 5 gives numerical results for a set of real data coming from bus and train schedules in the Montréal area. Section 6 concludes this paper.

Measuring synchronization

A great deal of research has been devoted to developing models and solution methods for timetable synchronization. In most research papers, the objective function consists of minimizing the total passenger waiting time, which can be derived from scheduled trips in the timetable. In [5], the objective function is to minimize the sum of all waiting times weighted by the number of passengers transferring. In [3], the objective function consists of maximizing the number of simultaneous arrivals of vehicles at connecting stops. This section proposes a more elaborate measure of synchronization derived from discussions with various schedulers who have addressed this issue in practice. While this measure can be used as an independent value that summarizes the synchronization of a timetable, we believe it should also be part of a global objective function designed to optimize a vehicle schedule (see section 4). Also note that although a stochastic component is also included in some published models (see [1], [6]), it is not considered in this paper.

Discussions with schedulers led us to identify two main factors that should be considered when evaluating the synchronization of a timetable:

- Waiting time between transfer places: the wait should neither be so short that some passengers are unable to make the connection, nor so long that passengers experience inconvenient delays. In fact, we found that schedulers can usually provide values for the minimum, ideal, and maximum waiting times for each pair of connecting places. These values normally depend on walking distances and passenger flow (possibly causing congestion) at specific transfer places and times.
- Relative importance of transfers: regardless of waiting time, some connections are to be favored. The number of involved passengers is an obvious element to consider, but not the only one. For instance, it is often particularly important to ensure a connection for the last trips of some routes, so passengers traveling late have a chance to reach their final destination. In some cases, it may also be desirable to preserve transfers that passengers have come to expect because they have existed for a long time.

The remainder of this section attempts to describe a synchronization measure that can take these factors into account.

Basic concepts

In our model, we use the concept of “trip meet” to describe a possible connection between two trips at a transfer place. For each trip meet, minimum, ideal, and maximum passenger wait times are specified. A range between three and ten minutes is typical, but this depends on the reliability of the routes involved, the estimated walking times at transfer points, and other factors such as trip frequencies. Meets are also defined with a weight factor that indicates their relative importance. This weight factor is generally strongly related to the estimated number of passengers involved in the connection, but other factors may also be involved.

Meets can be feasible or unfeasible. Feasible meets synchronize trips at a common timing point during the admissible passenger wait interval. A meet is usually associated to an “On trip” (the trip that passengers are transferring from) and a “Related trip” (the trip that transferring passengers are boarding). Meets are unfeasible if their times fall outside their admissible passenger wait interval, or if they have no trip to be synchronized with. As will be seen in the next section, unfeasible meets can be used to evaluate potential improvements to the synchronization of a timetable. They can also be used to detect historical feasible meets that become unfeasible due to changes in a timetable.

Figure 1 illustrates how some of these values can be graphically displayed, while Figure 2 provides a specific example. In this example, the on trip starts at 5:55, ends at 6:50, and passes the transfer node at 6:27. The related trip starts at this common transfer node at 6:36 and ends at 7:05. The trip meet’s minimum, ideal, and maximum wait times are respectively 3, 6, and 15 minutes. The meet is feasible, as the actual passenger waiting time falls within the admissible wait interval.

Trip meets can be specifically defined for a pair of trips, but it is generally more convenient to define meets for sets of trips that share some common features. To do so, we use the concept of “Meet builders”. Meet builders can be used to generate several meets for connections with specified characteristics. They typically include criteria describing the trips to consider (route, direction, time interval of the day), a meet place, values for the minimum, maximum, and ideal passenger wait times, as well as a weight factor assessing the relative importance of synchronization in this context. Once a meet builder is defined, trip meets can be generated for all possible connections that satisfy the detailed requirements.

In practice, meet builders are generally defined around peak hours. For instance, in large cities, most passengers commute from the suburbs to the downtown area (inbound) in the morning, and in the opposite direction (outbound) during the evening. Meet builders reflecting this reality are illustrated by the examples in Table 1. Meet builder 1 applies to connections between morning inbound trips on route 100 and 200 at place A. Since the passenger flow is estimated to be high in this context, a weight factor of 10 is specified for trip meets produced from this builder. Meet builder 2 applies to connections for the same routes, transfer place, and time interval, but in the opposite direction (outbound), which is known to involve fewer passengers at this time of day. The weight factor assigned to meet builder 2 is thus much lower than for meet builder 1. Meet builders 3 and 4 illustrate a similar situation for connections occurring during the afternoon peak hours, when the relative passenger flow is normally reversed to be in the opposite directions.

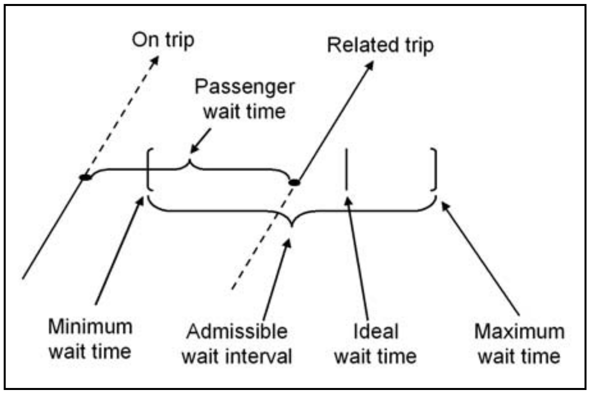


Figure 1 – Some values related to a trip meet.

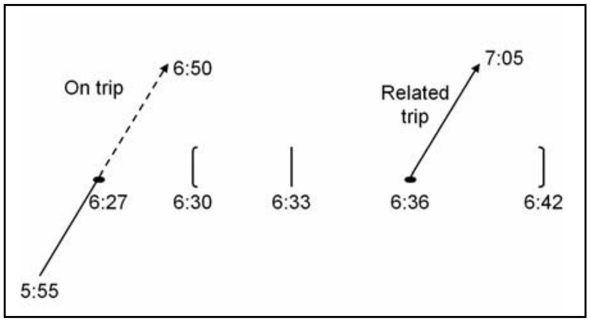


Figure 2 – A feasible trip meet.

Meet builder	On trips					Related trips		Weight factor
	Route	Direction	Transfer place	Start time	End time	Route	Direction	
1	100	Inbound	A	6:00	9:00	200	Inbound	10
2	200	Outbound	A	6:00	9:00	100	Outbound	2
3	100	Inbound	A	15:00	18:00	200	Inbound	2
4	200	Outbound	A	15:00	18:00	100	Outbound	10

Table 1 – Example of meet builders defined around peak hours.

A quantitative measure of synchronization

In order to develop an optimization method, we need an objective function that can summarize the synchronization level of a timetable. This function should consider the number of feasible meets in the timetable, the transfer waiting times, discourage the loss of predetermined historical meets, and consider weight factors associated to each trip meet. As a basis for such a measure, we introduce a synchronization quality index value QI_m for individual trip meets. This quality index is defined to yield higher values when the actual waiting time of a trip meet is close to its specified ideal wait time, and when the meet's specified weight factor is larger. A global timetable synchronization measure SQI can then be obtained by summing the quality indexes of all meets in a schedule. The following notation is used to provide a more complete description.

B is the set of all meet builders.
H the set of all historical meets.
M_b the set of all meets associated to meet builder b,
$I_{\min}$ is the minimum quality index base value for feasible meets.
$I_{\max}$ is the maximum quality index base value for feasible meets.
w_m is the weight factor associated to meet m,
a_m is the actual wait time of meet m,
l_m is the minimum wait time for meet m,
u_m is the maximum wait time for meet m,
i_m is the ideal wait time for meet m,
$InfeasCost$ is the cost for historical unfeasible meets.

The synchronization quality index value of meet m can then be defined as follows:

$$QI_m = \begin{cases} QI_m = w_m (I_{\min} + ((a_m - l_m)/(i_m - l_m))(I_{\max} - I_{\min})), a_m \in [l_m, i_m] \\ QI_m = w_m (I_{\max} + ((i_m - a_m)/(u_m - i_m))(I_{\max} - I_{\min})), a_m \in (i_m, u_m] \\ QI_m = 0, a_m \notin [l_m, u_m], m \notin H \\ QI_m = InfeasCost, a_m \notin [l_m, u_m], m \in H, \end{cases}$$

The global synchronization quality index of the timetable is defined as:

$$SQI = \sum_{b \in B} \sum_{m \in M_b} QI_m + \sum_{m \in H} QI_m .$$

The quality index of individual meets QI_m is thus defined as a piecewise linear function that returns proportional values between $w_m I_{\min}$ and $w_m I_{\max}$ according to the difference between the meet's actual wait time and its admissible bound values. For unfeasible meets, QI_m is also defined to be equal to the negative constant value $InfeasCost$ for historical meets, and zero otherwise (see Figure 3). The global synchronization quality index SQI thus incorporates all elements that have been previously discussed. Parameters $I_{\min}$, $I_{\max}$, and $InfeasCost$ can be set in order to reflect the relative importance of different factors. For instance, higher values for the ratio $I_{\min} / I_{\max}$ will yield greater quality index values for solutions that include more feasible meets, compared to lower ratio values that would promote trip meets with actual wait times that are close to their ideal values. Values of $I_{\min} = 7, I_{\max} = 10,$ and $UnfeasCost = -60$ have been used in the tests reported in Section 5, but can easily be changed to indicate different priorities. The quality index function could also easily be changed to include quadratic or other non-linear terms if it would be considered necessary to properly reflect requirements.

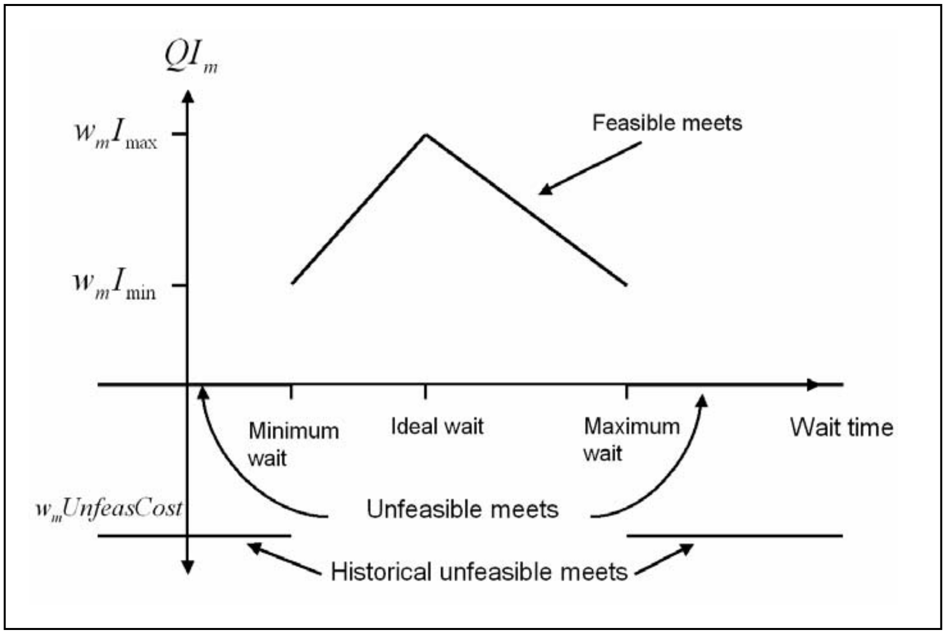
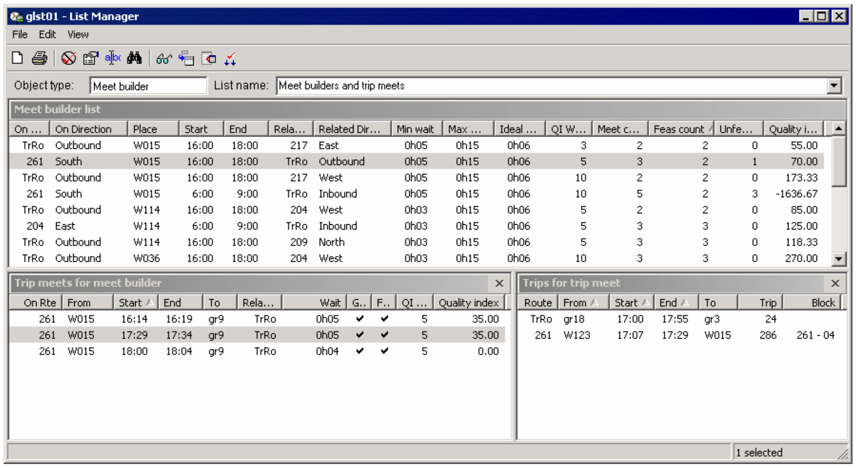


Figure 3 – Synchronization quality index of individual meets QI_m *as a function of wait time.*

Interactive tools

Although optimization algorithms can definitely be of great assistance for automatically synchronizing a timetable (see Section 4), it is also helpful to have access to powerful computerized interactive tools that can help analyze and improve solutions. In the *HASTUS* system, trip meets and meet builders are implemented as objects that can be easily visualized and manipulated. For instance, trip meets and meet builders can be displayed in different graphical views and lists that help schedulers recognize possible improvements.

The example in Figure 4 shows a list of meet builders with related lists showing meets and trips. When a meet builder is selected in the top list, all the meets that fulfill the builder's criteria are displayed in a second list (bottom left). Similarly, when trip meets are selected in the second list (bottom right), the trips involved are displayed.



Object type: Meet builder

List name: Meet builders and trip meets

Meet builder list

On ...	On Direction	Place	Start	End	Rela...	Related Dir...	Min wait	Max ...	Ideal ...	QI W...	Meet c...	Feas count	Unfe...	Quality i...
TrRo	Outbound	W015	16:00	18:00	217	East	0h05	0h15	0h06	3	2	2	0	55.00
261	South	W015	16:00	18:00	TrRo	Outbound	0h05	0h15	0h06	5	3	2	1	70.00
TrRo	Outbound	W015	16:00	18:00	217	West	0h05	0h15	0h06	10	2	2	0	173.33
261	South	W015	6:00	9:00	TrRo	Inbound	0h05	0h15	0h06	10	5	2	3	-1636.67
TrRo	Outbound	W114	16:00	18:00	204	West	0h03	0h15	0h06	5	2	2	0	85.00
204	East	W114	6:00	9:00	TrRo	Inbound	0h03	0h15	0h06	5	3	3	0	125.00
TrRo	Outbound	W114	16:00	18:00	209	North	0h03	0h15	0h06	5	3	3	0	118.33
TrRo	Outbound	W036	16:00	18:00	204	West	0h03	0h15	0h06	10	3	3	0	270.00

Trip meets for meet builder

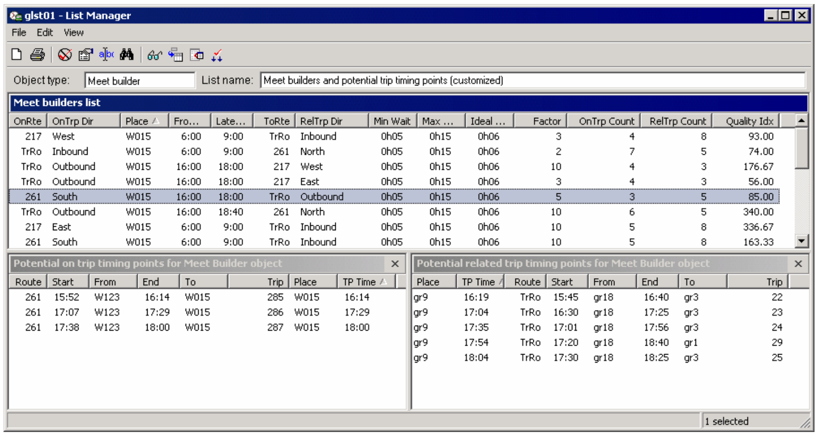
On Rte	From	Start	End	To	Rela...	Wait	G..	F..	QI ...	Quality index
261	W015	16:14	16:19	gr9	TrRo	0h05	✓	✓	5	35.00
261	W015	17:29	17:34	gr9	TrRo	0h05	✓	✓	5	35.00
261	W015	18:00	18:04	gr9	TrRo	0h04	✓	✓	5	0.00

Trips for trip meet

Route	From	Start	End	To	Trip	Block
TrRo	gr18	17:00	17:55	gr3	24	
261	W123	17:07	17:29	W015	286	261 - 04

1 selected

Figure 4 – Lists of meet builders, trip meets, and trips.



Object type: Meet builder

List name: Meet builders and potential trip timing points (customized)

Meet builders list

OnRte	OnTrp Dir	Place	Fro...	Late...	ToRte	RelTrp Dir	Min Wait	Max ...	Ideal ...	Factor	OnTrp Count	RelTrp Count	Quality Idx
217	West	W015	6:00	9:00	TrRo	Inbound	0h05	0h15	0h06	3	4	8	93.00
TrRo	Inbound	W015	6:00	9:00	261	North	0h05	0h15	0h06	2	7	5	74.00
TrRo	Outbound	W015	16:00	18:00	217	West	0h05	0h15	0h06	10	4	3	176.67
TrRo	Outbound	W015	16:00	18:00	217	East	0h05	0h15	0h06	3	4	3	56.00
261	South	W015	16:00	18:00	TrRo	Outbound	0h05	0h15	0h06	5	3	5	85.00
TrRo	Outbound	W015	16:00	18:40	261	North	0h05	0h15	0h06	10	6	5	340.00
217	East	W015	6:00	9:00	TrRo	Inbound	0h05	0h15	0h06	10	5	8	336.67
261	South	W015	6:00	9:00	TrRo	Inbound	0h05	0h15	0h06	10	5	8	163.33

Potential on trip timing points for Meet Builder object

Route	Start	From	End	To	Trip	Place	TP Time
261	15:52	W123	16:14	W015	285	W015	16:14
261	17:07	W123	17:29	W015	286	W015	17:29
261	17:38	W123	18:00	W015	287	W015	18:00

Potential related trip timing points for Meet Builder object

Place	TP Time	Route	Start	From	End	To	Trip
gr9	16:19	TrRo	15:45	gr18	16:40	gr3	22
gr9	17:04	TrRo	16:30	gr18	17:25	gr3	23
gr9	17:35	TrRo	17:01	gr18	17:56	gr3	24
gr9	17:54	TrRo	17:20	gr18	18:40	gr1	29
gr9	18:04	TrRo	17:30	gr18	18:25	gr3	25

1 selected

Figure 5 – Lists of meet builders, potential on trips, and potential related trips.

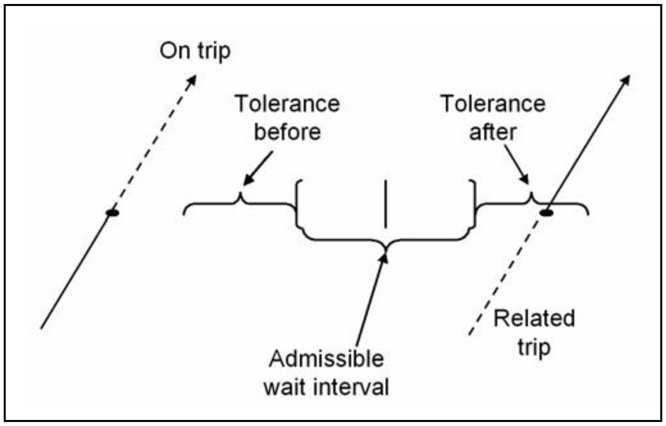


Figure 6 – Tolerance on passenger wait time.

Schedulers can reconfigure lists to display information in formats that are most convenient for a given task. For instance, Figure 5 shows a list of meet builders with related lists showing potential "on" and "related" trips. In this configuration, the related lists show trips with characteristics specified in a selected builder, even if their passing times are outside the admissible passenger wait interval. Schedulers can then easily detect potential meets that could be obtained by shifting candidate trips by a few minutes.

Tools are also available to automatically generate unfeasible meets. For instance, it is possible to specify tolerances to find qualified trips that meet a specified number of minutes before or after admissible passenger wait intervals (see Figure 6). Schedulers can then select the unfeasible meets to find missed transfers and produce connections with only minor changes to the timetable.

Other options can be specified to generate unfeasible meets for qualified trips that have no trip to meet with (one-trip meets). These options are typically used to evaluate the viability of trips on feeder routes. For instance, if bus routes are dedicated to bringing passengers to and from a train station, schedulers can generate one-trip meets to spot trips that should be moved or deleted because they have no train to connect with.

Optimization

The synchronization measure described in Section 2 can be integrated in an optimization algorithm that attempts to maximize the number of possible transfers in a timetable. Our experience and discussions with many schedulers have, however, made it clear to us that an effective approach should also consider other factors that impact the quality of a vehicle schedule. The factors considered in our method include:

- Maximizing the synchronization of the timetable.
- Minimizing the loss of some specified historical trip meets.
- Minimizing the number of necessary vehicles and blocks.
- Minimizing unproductive time (deadhead and layover).
- Minimizing deviations to planned headway (time intervals between consecutive departures on the same route/direction).
- Minimizing various penalties from operating rules and constraints.

Improving the synchronization of transfers in an existing solution implies that the initial trip timetable must be modified. Possible changes may consist of adding or removing some trips, reducing or increasing the duration of some trips, and/or shifting the planned start time of some trips. Our optimization algorithm only incorporates the latter approach, although all other types of modifications are available to schedulers in interactive mode.

Vehicle scheduling problems with trip shifting have been studied for some time and an overview of possible models can be found in [4]. Our existing scheduling optimization method already supports automatic trip shifting, as well as the usual decisions of linking trips into blocks and assigning blocks to depots and vehicle groups. Basically, our approach uses an algorithm that includes solving multiple network flow problems, lagrangean relaxation, as well as several heuristic mechanisms designed to efficiently generate high quality solutions. Our recent developments have focused on integrating synchronization into our cost structure, while maintaining an efficient implementation.

In our approach it is also important for the scheduler to specify penalty factors that indicate how the various objectives should be weighted against each other. Accordingly, all components of the objective function are expressed in minutes, and factors parameters are set to reflect the scheduler's preferences. For instance, some parameter values may lead the optimization algorithm to generate solutions that include a better synchronization, but require more vehicles. If this result is not acceptable to the scheduler, parameter values may be changed to increase the relative cost of vehicles.

Results

We tested our algorithm on a set of real data from the City of Montréal, Canada. In this context, the primary goal was to improve synchronization between bus and train routes used by passengers that commute between a suburb and the downtown area. Discussions with schedulers led us to define a set of meet builders that reflect the synchronization priorities to consider. As can be expected, meet builders with a higher weight factor were specified during the peak hours according to the natural flow of passengers. A number of historical trip meets were also defined to reflect the desire to preserve transfer opportunities that have been available for a long time. In our opinion, the need for preserving specific meets in the planning process will become less important in the future, as improved synchronization and increased access to passenger information tools should raise confidence in the availability of efficient transfer opportunities.

	Vehicle count	**Unproductive time**	**Meet count**	**Average deviation to ideal wait time**
Manual solution	10	17h30	64	0h03
Scenario 1	10	18h24	74	0h03
Scenario 2	11	18h57	85	0h01

Table 2 – Examples of possible scenarios.

Tests indicate that our optimization algorithm is very efficient and is able to consider several alternative solutions. Table 2 shows some results obtained on a set of about 300 trips from four bus routes that cross two train routes at five possible transfer points. The manual solution provided to us requires ten vehicles and includes 64 trip meets. Optimizing the timetable and vehicle schedule generated a solution (scenario 1) that added ten feasible meets by increasing unproductive time (layovers and deadheads) by 56 minutes. Setting the synchronization parameter to a very high value produces a solution (scenario 2) with 85 feasible meets, at the expense of an extra vehicle and a further increase in unproductive time.

The average deviation to the ideal wait time was three minutes in both the manual solution and scenario 1, and only one minute for scenario 2. Since these solutions can be generated rather quickly (usually less than five minutes on a Pentium 4 2Ghz processor for this data set), several scenarios can be produced using different parameter values, so the most appropriate solution can be selected.

Conclusion

This paper describes a set of tools that can be used to specify, measure, analyze, and optimize synchronization of a trip timetable. These tools can help produce cost-effective vehicle schedules that balance the need for efficient use of resources and high quality passenger services. It is our belief that in the long run, everybody will benefit from such efforts.

Acknowledgments

The authors would like to thank both the AMT (Agence Métropolitaine de Transport) and STM (Société de Transport de Montréal) for their financial support and active involvement in this research.

References

[1] Adamski, A., (1995) Transfer Optimization in Public Transport. In: Daduna JR, Branco I, Paixao JMP (eds) Computer-Aided Scheduling of Public Transport, Lecture Notes in Economics and Mathematical Systems 430, Springer, Berlin, pp. 23-38

[2] Ceder, A., (2001) Efficient Timetabling and Vehicle Scheduling for Public Transport. In: Voss S, Daduna JR (eds), Computer-Aided Scheduling of Public Transport, Lecture Notes in Economics and Mathematical Systems 505, Springer, Berlin, pp. 37-52.

[3] Ceder, A., Tal O (1999) Timetable Synchronization for Buses. In: Wilson NHM (ed), Computer-Aided Transit Scheduling, Lecture Notes in Economics and Mathematical Systems 471, Springer, Berlin, pp. 245-258.

[4] Daduna JR, Paixao JMP, (1995). Vehicle Scheduling for Public Mass Transit. In: Daduna JR, Branco I, Paixao JMP (eds), Computer-Aided Scheduling of Public Transport, Lecture Notes in Economics and Mathematical Systems 430, Springer, Berlin, pp. 76-90.

[5] Daduna JR, Voss S, (1995). Practical Experiences in Schedule Synchronization. In: Daduna JR, Branco I, Paixao JMP (eds), Computer-Aided Scheduling of Public Transport, Lecture Notes in Economics and Mathematical Systems 430, Springer, Berlin, pp. 39-55.

[6] Desilets A., Rousseau, JM., (1992). SYNCRO: A Computer-Assisted Tool for the Synchronization of Transfers in Public Transit Networks. In: Desrochers M, Rousseau JM (eds), Computer-Aided Scheduling of Public Transport, Springer-Verlag, pp. 153-166.