Integrated timetabling and vehicle scheduling

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Introduction

A public transit company's planning process generally involves many steps that include: network design, frequency setting, timetabling, vehicle scheduling, crew scheduling, and rostering. To reduce complexity, these tasks are often tackled sequentially, with each step providing the starting point for the next. However, it is possible to obtain significant savings when multiple steps are combined and optimized simultaneously. A previous paper (Integrated vehicle and crew scheduling in practice) explained vehicle and crew scheduling can be combined. This paper discusses the integration of timetabling and vehicle scheduling, and focuses on the main issues that must be addressed in order to reach viable solutions, as implemented in the *NetPlan* module.

An integrated timetabling and vehicle scheduling model

Consider a set I containing the trips in a timetable. For each trip $i \in I$, we define a set J_i that represents some key timing points of trip i. We can then introduce integer variables in which x_{ij} corresponds to the scheduled passing times of trip i at timing point $j \in J_i$. We want to define an optimization problem over this set of variables and we note that there are many aspects that have to be considered when evaluating a given solution x. We propose using the objective function $K_T T(x) + K_V V(x)$, which is divided into two main components.

The first part uses an evaluation $T(x)$ of the timetable that does not consider the costs that will eventually be incurred when the trips will be covered by a vehicle schedule (the "pure" timetabling problem). The second part represents the vehicle cost $V(x)$ that can be obtained by solving a vehicle scheduling optimization problem over the trips defined by timetable x. Parameters K_T and K_V represent weights that specify the relative importance given to each component. Using this notation, we can define problem (P) as described below (additional constraints on the x variables could also be inserted as needed to describe specific requirements for the timetable).

$$\begin{array}{lll} \text{Min} & K_T T(x) + K_V V(x) & \text{(P)} \\ \text{s.t.} & x_{ij} \text{ integer } \ \forall i \in I, \forall j \in J_i & \end{array}$$

In the context of rail operations, function $T(x)$ can heavily penalize violations to hard constraints that prevent collisions for trips that share a common infrastructure. By contrast, these restrictions are usually formulated as direct constraints in the Periodic Event Scheduling Problem (PESP) [3], but we prefer to move them to the objective to simplify the description of our model. In practice, $T(x)$ can also capture several attributes that measure the quality of timetable x from the viewpoint of travelers. For instance:

- A timetable should provide an efficient offering to passengers that travel using the transit network. With fixed frequencies, this performance can often be estimated by evaluating the waiting times at key connections where passengers transfer between lines.
- A timetable should lead to reliable and punctual operations. These objectives are typically controlled by the running and loading times that are allocated to each trip.
- A timetable should provide a regular headway between trips.

In practice, $V(x)$ can be roughly estimated by a formula, or evaluated more precisely by solving a vehicle scheduling problem. This task can be rather easy in simple cases (e.g. one depot and vehicle type), or constitute a difficult challenge for the more complex variants (e.g. multiple depots and vehicle types). In some rail contexts, the movements of vehicles are so constrained by the infrastructure that the timetabling and vehicle scheduling components become closely tied together. Specific linear constraints that relate to admissible vehicle links can then be directly added in the PESP framework. With bus operations, vehicles can move more freely on the network, which leads to many more possible trip links between different lines.

Solution methods

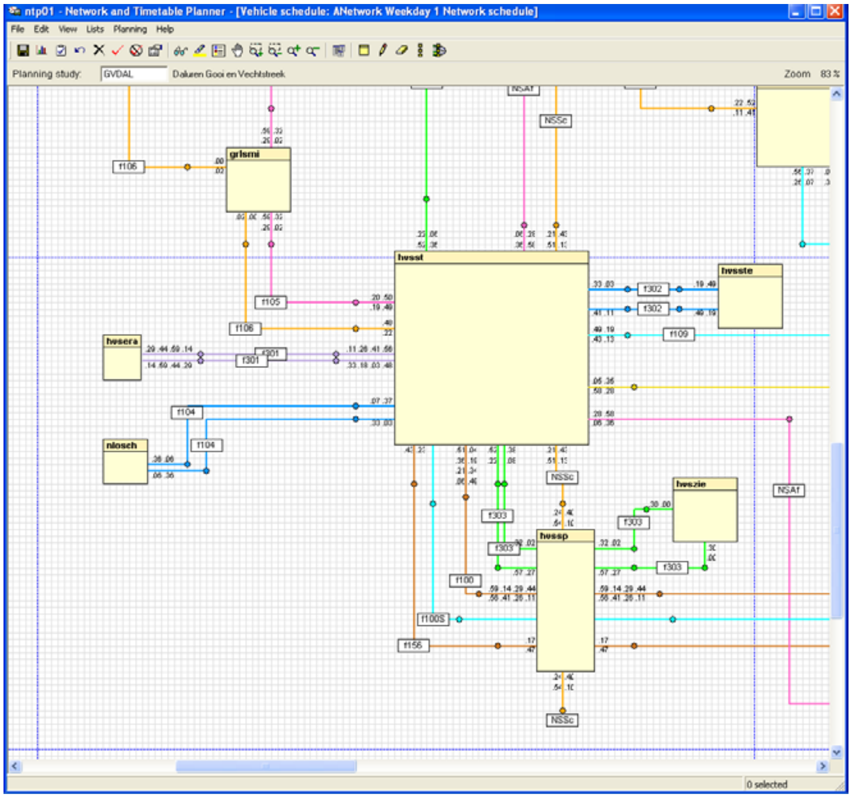
Several strategies can be adapted to heuristically solve problem (P), and the preferred approach will depend on the objectives of the planners. In practice, $T(x)$ can generally be evaluated relatively efficiently. For instance, synchronization can be estimated by a function that counts the number of possible transfers for passengers. The evaluation of $V(x)$ is much more costly since it involves solving an auxiliary optimization problem. A planner is also often forced to reach an acceptable compromise between passenger comfort and efficient use of available resources. Parameter weights K_S and K_V can be varied according to the user's preference. If $K_V = 0$, the vehicle costs are ignored, which may give rise to a quadratic assignment problem similar to the one that is reported in [1]. If both K_S and K_V are non zero, (P) becomes a very difficult problem to solve to optimality.

GIRO has developed several heuristic approaches that essentially use local search methods defined over neighbourhoods of variables x_{ij}.

Neighbourhoods may include changing the values of one or more variable at a time, and using some techniques that attempt to identify promising search directions on specific trips. For very large problems, solution times can be decreased by solving a relaxation of $V(x)$ at each iteration. We can also optimize on a selected or limited time frame, which can still allow identifying good partial timetables that can be cyclically applied over a longer period.

Integration into a decision support system

As with any optimization algorithm that must be used by practitioners, it is essential to provide users with interactive tools that enable them to visualize and manipulate the generated solutions. The *NetPlan* module included in GIRO's *HASTUS* software solution has been specifically developed to facilitate the integration of the timetabling and vehicle scheduling processes in practice. This module was originally developed in collaboration with the Connexxion group in Holland, and has recently been implemented in two other large European public transport companies. The following diagram gives an example of the graphical display of a partial timetable in *NetPlan*.



Planning study: GVDAL Daluren Gooi en Vechtstreek

References

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